day 29

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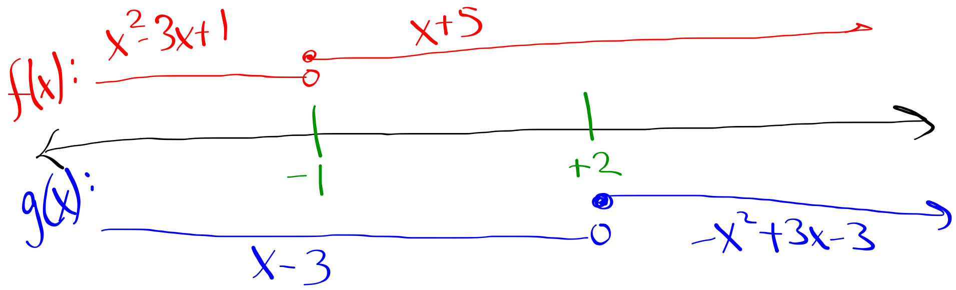
day 29

12·17·16·5·14·1·8·13·23)

(12) Let $f(x) = \begin{cases} x^2-3x+1, & x<-1 \\ x+5, & x\geq -1 \end{cases}$

$g(x): \begin{cases} x-3, & x<2 \\ -x^2+3x-3, & x\geq 2 \end{cases}$



$$(f+g)(x) = \begin{cases} x^2-3x+1+(x-3), & \text{if } x<-1 \\ x+5+(x-3), & \text{if } -1\leq x<2 \\ x+5+(-x^2+3x-3), & \text{if } x\geq 2 \end{cases}$$

−1

i) $f+g(-1)$ exists

ii) $\lim_{x\to -1^-} f+g(x) = \lim_{x\to -1^-} x^2-2x-2$

$= (-1)^2-2(-1)-2 = 1$

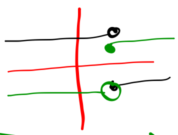
$\lim_{x\to -1^+} f+g(x) = \lim_{x\to -1^+} 2x+2 = 0$

uh oh No match

$\emptyset$

If f(x) is not continuous, Then f+g(x) is not continuous

FALSE



If f(x) is not continuous, AND g(x) is continuous, then f+g is NOT continuous.

TRUE, but not something we "know"

2

i) $f+g(2)$ exists

ii) $\lim_{x\to 2^-} f+g(x) =$

$\lim_{x\to 2^-} 2x+2 = 6$

$\lim_{x\to 2^+} f+g(x) = \lim_{x\to 2^+} -x^2+4x+2$

$= -(2)^2+4(2)+2 = 6$

MATCH!

$\lim_{x\to 2} f+g(x) = 6$

iii) $f+g(2) = 6 = \lim_{x\to 2} f+g(x)$

$\therefore$ CONTINUOUS!

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(17) $\lim_{x\to\infty} \frac{\sqrt{x^2+3}-x}{1} \cdot \frac{\sqrt{x^2+3}+x}{\sqrt{x^2+3}+x} = \lim_{x\to\infty} \frac{(x^2+3)-x^2}{\sqrt{x^2+3}+x}$

$= \lim_{x\to\infty} \frac{3}{\sqrt{x^2+3}+x} = 0$

what about $\lim_{x\to\infty} \underbrace{x}_{\infty} \left(\underbrace{\sqrt{1+\frac{3}{x^2}}-1}_{0}\right)$

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$\boxed{16}$ ok.

Real #s

rational numbers

irrational numbers

$$f(x) = \begin{cases} 0 & \text{if } x \text{ rational} \\ 2x & \text{if } x \text{ irrational} \end{cases}$$

is $f(x)$ cont at $x=0$?

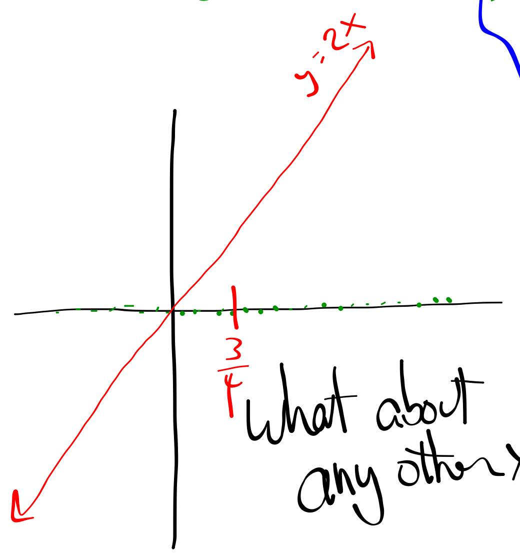
$0 \le f(x) \le 2x$

for every x in $\mathbb{R}$

$$\lim_{x\to 0} 0 \le \lim_{x\to 0} f(x) \le \lim_{x\to 0} 2x$$

$\quad\| \qquad\qquad ? \qquad\qquad \|$

$\quad 0 \qquad\qquad\qquad\qquad 0$



what about any other $x \neq 0$?

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5)

$$\lim_{x\to-\infty} \frac{10x^3-3x^2+8}{\sqrt{25x^6+x^4+2}} = \lim_{x\to-\infty} \frac{x^3\left(10-\frac{3}{x}+\frac{8}{x^3}\right)}{\sqrt{x^6}\sqrt{25+\frac{1}{x^2}+\frac{2}{x^6}}}$$

$$= \lim_{x\to-\infty} \frac{x^3\left(10-\frac{3}{x}+\frac{8}{x^3}\right)}{(-x^3)\left(\sqrt{25+\frac{1}{x^2}+\frac{2}{x^6}}\right)}$$

$$= \lim_{x\to-\infty} -\frac{10-\frac{3}{x}+\frac{8}{x^3}}{\sqrt{25+\frac{1}{x^2}+\frac{2}{x^6}}} = -\frac{10}{5} = -2$$

14) SPRE

$$\lim_{x\to 9} \frac{12(\sqrt{x}-3)}{x-9} \cdot \frac{\sqrt{x}+3}{\sqrt{x}+3}$$

$$= \lim_{x\to 9} \frac{12(x-9)}{(x-9)(\sqrt{x}+3)} = \lim_{x\to 9} \frac{12}{\sqrt{x}+3} = \frac{12}{3+3}$$

$$= \frac{12}{6} = 2$$

Oct 8-7:30 AM

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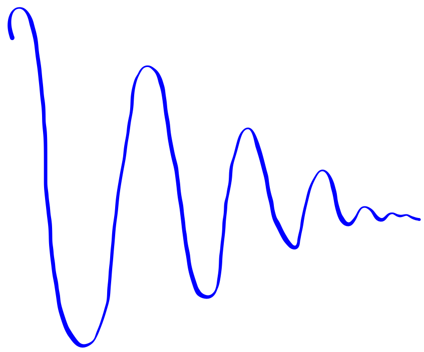
1) SORRY $\lim_{x\to\infty} \frac{\sin(2x)}{3x} = 0$



8) $\frac{x^2+4}{|x|^2-4} = \frac{x^2+4}{x^2-4}$

2 VA at $x=-2, 2$

1 HA at $y=1$

Oct 8-7:30 AM

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2) clue: $\frac{x^2-9}{3x-9} = \frac{(x-3)(x+3)}{3(x-3)}$

3) $\frac{\sin(x)}{x^2+3x}$ [[I lied... once]] $= \frac{\sin x}{x} \cdot \frac{1}{x+3}$

$\lim_{x\to 0} \left(\frac{\sin x}{x}\right)\left(\frac{1}{x+3}\right)$

$1 \cdot \frac{1}{3}$

take limit as $x \mapsto 0$

7) $\frac{x^2-x}{2x} = \frac{x(x-1)}{x(2)}$ so $\lim_{x\to 0} f(x) = \lim_{x\to 0} \frac{x(x-1)}{x(2)} = \lim_{x\to 0} \frac{x-1}{2} = -\frac{1}{2}$